Differences Between Curves in Computer Graphics and Their Rendering

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ABSTRACT The aim of this paper is to give a definition of different curves that are used in computer graphics and the method of their rendering. We will give a mathematic describe of different curves and their main characteristic. We will describe several techniques commonly used for rendering curves.

Keywords – Curves, Surfaces, Implicit curves, Explicit curves, Parametric curves, Bezier curves, B-spline curves, Rendering.

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1. INTRODUCTION

Computer graphics are used to simplify display a picture of any size on a computer screen. To generate graphics in computers various algorithms and techniques are used. In computer graphics, we often need to draw different types of objects onto the screen. Objects are not flat all the time and we need to draw curves many times to draw an object. There are many types of curves used in computer graphics.

In computer graphics, curves and surfaces are fundamental concepts used to represent and render objects in a virtual environment.

Curves are one-dimensional representations that are defined by a set of control points and mathematical equations. They are commonly used to represent smooth and continuous shapes, such as lines, arcs, and splines. Curves can be rendered using various algorithms, such as Bézier curves or B-splines, which interpolate the control points to create a smooth curve.

On the other hand, surfaces are two-dimensional representations that define the shape and geometry of objects in computer graphics. They are typically composed of a network of curves or a set of control points that define the surface's shape. Surfaces can be used to represent complex objects like 3D models, terrain, or character meshes. Rendering surfaces involves techniques like polygonal mesh rendering, where the surface is approximated by a collection of polygons, or more advanced methods like ray tracing or physically-based rendering.

The main difference between curves and surfaces lies in their dimensionality. Curves exist in one dimension and are used to represent linear or curved shapes, while surfaces exist in two dimensions and are used to represent more complex and volumetric objects. Additionally, the rendering techniques for curves and surfaces may differ, as curves can be rendered using interpolation algorithms, while surfaces often require more complex rendering algorithms to accurately represent their geometry and appearance.

It's important to note that both curves and surfaces play crucial roles in computer graphics, and their rendering techniques continue to evolve with advancements in technology and algorithms.

A curve is an infinitely large set of points. Each point has two neighbors except endpoints. Curves can be broadly

classified into several categories:
- implicit
- explicit
- parametric curves
- Bezier curves
- B-spline curves

2. IMPLICIT CURVES

An implicit curve or surface is the set of zeros of a function of 2 or 3 variables. Implicit curve functions are used to define lines and planes. Provides no control over tangents at connection points when joining several implicit functions. Implicit functions are hard to find for many shapes. Use a function that states which points are on and off the curves.

The definition of implicit curves:
- All lines are defined with: Ax+By+C=0
- A surface in three dimensions f (X, Y, Z) is define with:
- Any plane Ax+By+Cz+D=0, with constants a,b,c, and d.
- A sphere centered at the origin with a radius r: $x^2 + y^2 + z^2 - r^2 = 0$
- Implicit functional form is define with: f(x,y) = 0

3. EXPLICIT CURVES

Explicit curves are single value. Mathematical function is y = f(x) can be plotted as curve. This function is explicit representation of curve. The main characteristics of explicit curves are:
- Do not allow for multiple values for a given argument
- Cannot describe vertical tangents, as infinite slopes are hard to represent.
- Cannot represent all curves (vertical lines, circles)

4. PARAMETRIC CURVES

Curves have a parametric form called parametric curves. A curve in the plane is said to be parameterized if the set of coordinates on the curves (x,y,z) is represented as a function of a variable t. The variable t is called a parameter and the relations between x,y,z, and t are called a parametric equation:

$$Y^2 = 4abp(t) = f(t), g(t) \, or \, p(t) = x(t), y(t)$$

The parametric form of a curve is a function that assigns a position to values of the free parameters. That the parametric function is a vector-valued function. This example is a 2D curve, so the output of the function is a 2-D vector, in 3D it would be a 3 vector. It is simple and flexible.

The parametric form is suitable for representing closed and multivalued curves. In parametric curves, each coordinate of a point on a curve is represented as a function of a single parameter. There are many curves that we cannot write down as a single equation in terms of x and y. The position vector of a point on the curve is fixed by the value of the parameter. Since a point on a parametric curve is specified by a single value of the parameter, the parametric form is axis-dependent. The function of each coordinate can be defined independently:

$$(X(t), Y(t)) : 1D \, curve \, in \, 2D \, curve \, (X(t), Y(t), Z(t)) : 1D \, curve \, in \, 3D \, space$$
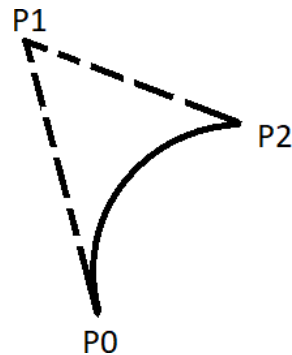
5. BEZIER CURVES

These curves are discovered by the French engineer Pierre Bezier. A Bezier curve is particularly a kind of spline generated from a set of control points by forming a set of polynomial functions. These functions are computed from the coordinates of the control points. These curves can be generated under the control of other points. Tangents by using control points are used to generate curves.

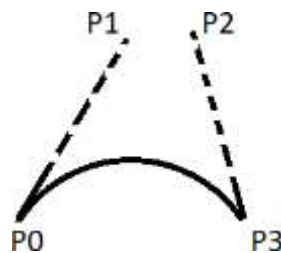Different types of curves are Simple, Quadratic, and Cubic.
- Simple Bezier curve is a straight line from the point.
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P1

B(1/2)

-     P0
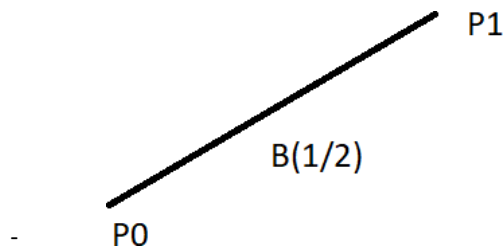
- Quadratic Bezier curve is determined by three control points.

P1

P2

P0

- The cubic Bezier curve is determined by four control points.

P1   P2

P0   P3

The main characteristics of Bezier curves are:

Bezier curves are widely available and used in various CAD systems, in general graphics packages such as GL

- The slope at beginning of the curve is along the line joining the first two control points and the slope at the end of the curve is along the line joining the last two points
- Bezier curve always passes through the first and last points i.e. p(o)=po, p(1, =pnlie)
- The curves lie entirely within the convex hall formed by the four control points
- The slope at the beginning of the curve is along the line joining the first two control points and the slope at the end of the curve is along the line joining the last two points.
- The degree of polynomial defining the curve segment is one less than the no of defining the polygon.

6. B-SPLINE CURVES

The Bezier-curve produced by the Bernstein basis function has limited flexibility. First, the number of specified polygon vertices fixes the order of the resulting polynomial which defines the curve. The second limiting characteristic is that the value of the blending function is nonzero for all parameter values over the entire curve.

The main characteristics of B-spline curves are:

The maximum order of the curve is equal to the number of vertices of defining polygon.

-        The degree of B-spline polynomial is independent on the number of vertices of defining polygon.
-        B-spline allows the local control over the curve surface because each vertex affects the shapeof a curve only over a range of parameter values where its associated basis function is nonzero.
-        The curve exhibits the variation diminishing property.
-        The curve generally follows the shape of defining polygon.
-        Any affine transformation can be applied to the curve by applying it to the vertices of defining polygon.
-        The curve line within the convex hull of its defining polygon.
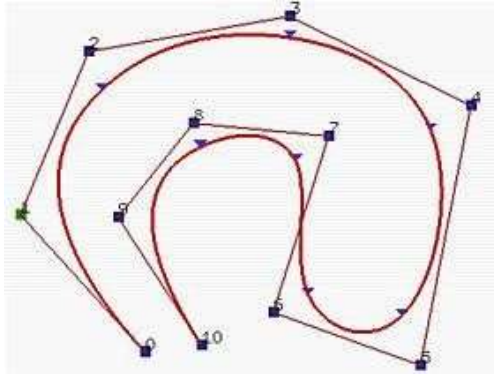
7. RENDERING DIFFERENT CURVES IN COMPUTER GRAPHIC

Rendering curves in computer graphics involves the process of converting mathematical representationsof curves into visual representations on a screen or in a rendered image. There are several techniques commonly used for rendering curves:

1.   Rasterization: This technique involves approximating curves by a series of short line segments, also known as line segments or line primitives. The curve is divided into small segments, and each segment is rendered as a straight line. Rasterization is a fast and efficient method commonly used in real-time graphics.

2.   Bézier Curves: Bézier curves are a popular mathematical representation for smooth curves. They are defined by control points that influence the shape of the curve. Rendering Bézier curves involves evaluating the curve equation at various points and connecting them to form a smooth curve. This technique is commonlyused in graphics software for creating and rendering curves.

3.   B-splines: B-splines are another mathematical representation for curves that provide more flexibility and control over the shape. B-splines are defined by control points and basis functions. Rendering B-splines involves evaluating the basis functions and control points to generate the curve. B-splines are commonly usedin computer-aided design (CAD) and animation software.

4.   Anti-aliasing: Curves can appear jagged or pixelated when rendered due to the discrete nature of digital displays. Anti-aliasing techniques are used to smooth out the edges of curves and reduce the appearance of aliasing artifacts. This can be achieved through techniques such as supersampling, where the curve is renderedat a higher resolution and then downsampled to the display resolution.

These are just a few examples of techniques used for rendering curves in computer graphics. The choice of technique depends on factors such as the desired level of detail, performance requirements, and the specific application or software being used.

Journal Papers:

8. REFERENCES

[1]   Farin, Gerald, *Curves and Surfaces for CAGD (Computer Aided Graphics and Design),* San Diego, Academic Press, (2001).

Books:

[2]   David J. Eck Hobart and William Smith Colleges*, Introduction to Computer Graphics,* Department of Mathematics and Computer Science, (2023).

[3]   Gerald Farin, *Curves and Surfaces for CAGD* (Arizona State University, 2002).

[4]   Beach, Robert C., *An Introduction to the Curves and Surfaces of Computer-Aided Design*, New York, Van Nostrand Reinhold, (1991).